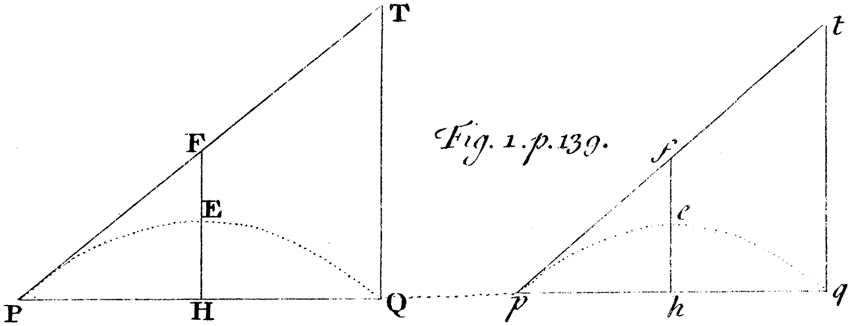


Fig. 1. p. 139.

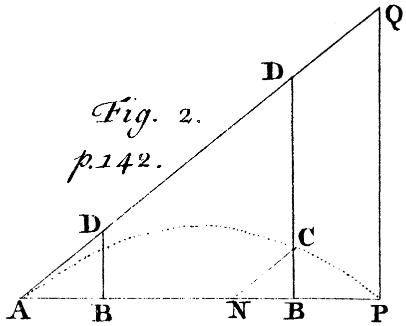


Fig. 2. p. 142.

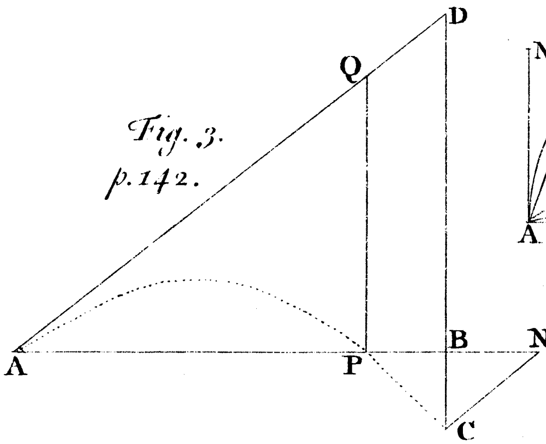


Fig. 3. p. 142.

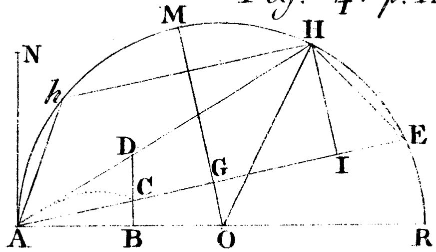


Fig. 4. p. 143.

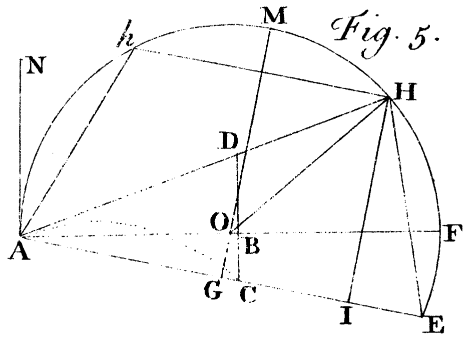


Fig. 5.

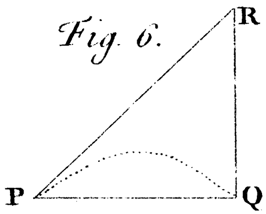


Fig. 6.

p. 146.

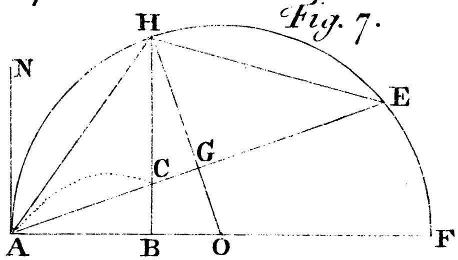


Fig. 7.

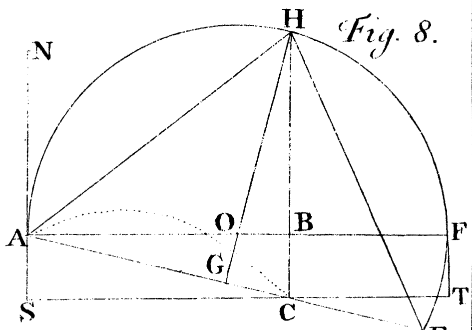
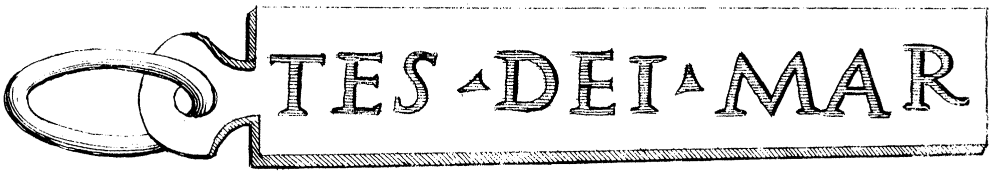
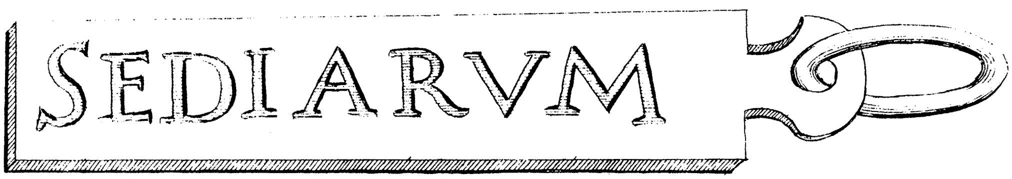


Fig. 8.



p. 225.



happen'd about the Time of the natural Birth, the Child then must have continued alive some considerable Time afterwards, during which these bony Excrescences were formed; there being a perfect Ossification, as performed by the Laws of Circulation, and not by any vegetative or petrifying Power, as in inanimate Bodies.

Two or three of the lateral Processes of the Spine were what first passed thro' the little Ulcer; the rest of the Bones (except a few that were lost in cleaning) were presented by the Doctor to the Museum of the *Royal Society*. They retain a very strong and singular Smell, though they were immediately cleansed from the rotten Flesh, and well washed.

The Woman came by Sea to *Stockholm* above a Year after this Cure, and was presented to the *Academy* in good Health; and the Doctor believes she is still alive and well.

II. *The Motion of* Projectiles *near the Earth's Surface consider'd, independent of the Properties of the Conic Sections; in a Letter to* Martin Folkes *Esquire, Pr. R. S. by Mr.* Tho. Simpson *F. R. S.*.

Read Feb. 4. 1747.

AFTER so much as has been already said upon the Motion of Projectiles *in vacuo*, it may seem needless to attempt any thing further on that Head; nevertheless, as a thorough Knowledge in the Art of Gunnery is become more than

than ever neceſſary, and as Gentlemen employ'd in the Practice of that Art are (I am ſenſible) too often deterr'd from applying themſelves to the Theory, by the Difficulties they imagine they ſhall meet with in the Conic Sections, you will, I hope, pardon the Liberty I have taken, in troubling you with my Thoughts on a Subject, in which little or nothing new is to be expected beſides the Method.

When I firſt drew up this Paper (which was about two Years ago) I did intend, had Health permitted me to make the proper Experiments, to have alſo attempted ſomething with reſpect to the Reſiſtance of the Atmoſphere, whereof the Effects are indeed too conſiderable to be intirely diſregarded: But if the Amplitude of the Projection, anſwering to one given Elevation, be firſt determined by Experiment (which our Method ſuppoſes) the Amplitudes in all other Caſes, where the Elevations and Velocities do not very much differ from the firſt, may be determined, by the Proportions here laid down, to a ſufficient Degree of Exactneſs: Becauſe, in all ſuch Caſes, the Effects of the Reſiſtance will be nearly as the Amplitudes themſelves; and were they accurately ſo, the Proportions of the Amplitudes, at different Elevations, would be exactly the ſame as *in vacuo*; which Proportions I now proceed to determine.

PROBLEM I.

Let two Balls be projected with the ſame Celerity, at different, but given Elevations, 'tis propoſed to determine the Ratio of the Times of their Flight,

Flight, of their greatest Altitudes, and of their horizontal Amplitudes.

Let Pq (*Fig.* 1.) represent the Plane of the Horizon, PEQ and peq the Paths of the Projectiles, described in the Flight; moreover let QPT and qpt be the given Angles of Elevation, and let PQ and pq be bisected in H and h; drawing HE, QT, he and qt, all perpendicular to Pq; and making the Sine of $QPT = S$, its Co-sine $= C$, the Sine $qpt = s$, its Co-sine $= c$, and Radius $= r$.

Therefore, since the Distances descended by heavy Bodies (whether from a Point at Rest, or from the right Lines in which they *would* move, if not acted upon by Gravity) are known to be as the Squares of the Times, QT will be to qt, as the Square of the Time of describing PEQ (or of that wherein the Ball would move uniformly over the Space PT with its first Velocity at P) is to the Square of the Time of describing peq (or of that wherein the other Ball would move uniformly thro' the Length pt). But the Celerities at P and p being equal, by Hypothesis, the Times in which the said Lines PT and pt would be uniformly described, are manifestly, as the Lines themselves: Whence the Squares of those Lines must, also, be as the Squares of the Times, and, consequently, as the Distances descended: that is, $Pt^2 : pt^2 :: TQ : tq$.

Now, by Plane Trigonometry $TQ = \frac{S \times PT}{r}$ and $tq = \frac{s \times pt}{r}$; therefore $PT^2 : pt^2$ $\left(:: \frac{S \times PT}{r} : \frac{s \times pt}{r}\right) :: S \times PT : s \times pt$; whence, by dividing the Antecedents

 by

by PT, and the Consequents by pt, we have $PT.\ pt :: S : s$; from which it appears, that the Times of Flight are directly as the Sines of Elevation.

Again, the Times of describing EQ and eq (which are the Halves of the Wholes) being also to one another as $S : s$, and the Distances EH, eh descended in them, as the Squares of the Times, it likewise follows, that $S^2 : s^2 :: EH : eh$; or that the greatest Altitudes are as the Squares of the Sines of Elevation.

Moreover, because (by Trigonometry) $PT = \frac{r \times PQ}{C}$ and $pt = \frac{r \times pq}{c}$; and it has been already proved, that, $S : s :: PT : pt$, it follows, that $S : s :: \frac{r \times PQ}{C} : \frac{r \times pq}{c}$; whence, by multiplying the Antecedents by $\frac{2C}{r}$ and the Consequents by $\frac{2c}{r}$, it will be $\frac{2SC}{r} : \frac{2cs}{r}$ $(:: 2PQ : 2pq) :: PQ : pq$. But $\frac{2SC}{r}$ is known to be the Sine of double the Angle whose Sine is S, and Co-sine C, &c. Therefore the horizontal Amplitudes are to one another, as the Sines of the double Elevations.

Corol. 1.

Hence it follows, that the greatest Amplitude possible will be, when the Elevation is half a Right Angle, or 45 Degrees (because the Sine of 90° is the greatest of all others).

Corol.

Corol. 2.

Therefore, if the greateſt Amplitude be given (from Experiment) the Amplitude anſwering to any propoſed Elevation, above, or below, 45 Degrees, may from hence be found: For it will be as the Radius, to the Sine of double the given Elevation, ſo is the greateſt, to the required, Amplitude.

Corol. 3.

Hence, alſo, the Altitude of the Projection may be known; for QT, when the Angle QPT is half a Right Angle, will be $= PQ$; and therefore HE $(\frac{1}{4}TQ) = \frac{1}{4}PQ$; alſo, in this Caſe, $S^2 = \frac{1}{2}r^2$; whence our Proportion $S^2 : s^2 :: HE : he$ will here become $\frac{1}{2}r^2 : s^2 :: \frac{1}{4}PQ : he$; from whence it appears, that, as the Square of the Radius is to the Square of the Sine of any given Elevation, ſo is half the greateſt horizontal Amplitude, to the Altitude of the Projection. Hence it alſo follows, that the Height to which the Ball would aſcend, if projected directly upwards, is juſt half the greateſt Amplitude.

Corol. 4.

Therefore, ſince it is well known, that a Body *in vacuo* aſcends and deſcends with the ſame Velocity; and that the Diſtances deſcended are as the Squares of the Velocities; it follows, that the Amplitudes, at the ſame Elevation, with different Velocities, will alſo be to one another as the Squares of the Velocities; becauſe they are as the greateſt Amplitudes, with the ſame Velocities (by *Corol.* 2.)

and theſe are as the Diſtances perpendicularly deſcended (*by the precedent*). Whence, *univerſally*, if both the Elevations and the Velocities differ, the Amplitudes will be to each other in a *Ratio* compounded of the *Ratio's* of the Sines of double the Angles of Elevation, and of the duplicate *Ratio's* of the Velocities, or impelling Forces.

Problem II.

The Angle of Elevation, and the greateſt horizontal Amplitude, being given, to find at what Diſtance the Piece ought to be planted, to hit an Object, whoſe Diſtance, above or below the Plane of the Horizon, is alſo given.

Let AB (*Fig.* 2 and 3.) be the Plane of the Horizon, BC the perpendicular Height or Depreſſion of the Object, and AB the required Diſtance: Alſo let BC be produced to meet the Line of Direction AD in D, and let P be the Place where the Path of the Projectile would meet the Horizon; moreover, let PQ be perpendicular to AP, and CN parallel to AD. Then, by the preceding Problem, it will be as Radius : the Sine of $2BAD$ :: the given (or greateſt) Amplitude : AP; which therefore, is known.

Moreover, the Areas of ſimilar Triangles being as the Squares of their homologous Sides, we have $AP \times PQ : AB \times BD :: AQ^2 : AD^2$. But $AQ^2 : AD^2 :: AB \times BD :: QP : DC$ (from Principles already explained) therefore, by Equality, $AP \times PQ : AB \times BD :: QP : DC$; and conſequently $AP : AB :: BD : CD$; but (becauſe of the parallel Lines CN and AD) $BD : CD :: AB : AN$; whence, again

again by Equality, $AP : AB :: AB : AN$; therefore, by Division, $AP : BP :: AB : BN$; and, consequently $AP \times BN = BP \times AB$.

Let AP be now bisected in O; then $BP \times AB$ being $= AO^2 - OB^2$ (in the first Case) and $= OB^2 - AO^2$ (in the second Case), we shall therefore have $OB^2 = AO^2 \mp AP \times BN = AO \times \overline{AO \mp 2BN}$: whence the Distance AB is likewise known. *Q.E.I.*

Corollary.

Hence, if the Elevation, and the greatest Amplitude, together with the Distance AB of the Object be given, the Height or Depression of the Ball in the Perpendicular BCD will be known: For it is proved, that $AP : BP :: BA : BN$; whence BN is known: But, as the Radius to the Tangent of BNC (BAD): so is BN to BC.

Problem III.

The greatest horizontal Amplitudes of the Piece, together with the Distance and Height (or Depression) of the Object being given, to find the Direction or Angle of Elevation.

Let BC (*Fig.* 4 and 5.) be the perpendicular Height or Depression of the Object, AB its given horizontal Distance, and AH the required Direction; Also let PQ (*Fig.* 6.) be the greatest Amplitude (answering to 45° of Elevation); draw AC, in which produced (if need be) take $AG = PQ$; make MGO perpendicular to AG, meeting AB produced (if need be) in O; and from the Centre O, with the Interval OA,

let

let a Circle be described, intersecting AG, produced in E, and the Line of Direction AD in H; join E, H, and let HI, AN and QR, be perpendicular to AE, AO, and PQ respectively, and let BC, produced, meet AH in D.

It will appear, from what has been said above, that $AD^2 : PR^2 :: DC : RQ$; therefore PR^2 being $= 2PQ^2 = 2AG^2 = \frac{1}{2}AE^2$, and $RQ = PQ = \frac{1}{2}AE$ (by Construction), we have $AD^2 : \frac{1}{2}AE :: DC : \frac{1}{2}AE$, and therefore $AD^2 = AE \times DC$.

Now, the Triangles ADC, AEH, being equiangular (because $ADC = DAN = AEH$, and DAC common to both) we likewise have $AD : DC :: AE : EH$, and consequently $AE \times DC = AD \times EH = AD^2$ (*per* above); whence $EH = AD$. Therefore, as the Triangles ADB and EHI are equiangular, they are equal in all respects; and so $HI = AB$: Whence follows this easy

Construction.

Having described the Circle AEF, as above directed, and drawn MG perpendicular to AE, take Gn equal to AB, and thro' n, parallel to AE, draw Hh, cutting the Circle in H and h; join A, H, and A, h; then either of the Directions AH or Ah, will answer the Conditions of the Problem. From this Construction we have the following Calculation, *viz.*

As AB is to BC, so is AG to OG; which added to, or subtracted from, Gn (AB) gives On: Then, it will be, as $AG : On ::$ the Co-sine of OAG : Co-sine of HOn ($= HAh$) the Difference of the two

required

required Elevations; whence the Elevations themselves are known. *Q. E. I.*

Corol. 1.

Hence, if the Elevation of the Piece, with the Distance and the Height (or Depression) of the Object be given, the greatest horizontal Amplitude may be found: For it will be $AB : BC$: : Radius : Tang. of BAC; whence CAD is also known.

Then, $S. CAD : S. ACD$ (AHE) : : AD (HE) : AE.

And, $S. ADC$: Radius : : $AB : AD$.

Therefore, by compounding these Proportions, we have $S. CAD \times S. ADC$: Radius $\times S. ACD$: : $AB : AE$; which is equal to twice the required Amplitude, by Construction.

Corol. 2.

Moreover, if the Elevation, and the greatest horizontal Amplitude be given, the Amplitude of the Projection on any ascending or descending Plane AE, whose Inclination FAE is also given, may from hence be derived. For, $S. AHE$ (ACD) : $S : EAH$ (CAD) : : AE ($2PQ$) : EH (AD) and $S. ACD$: $S. ADC$: : $AD : AC$; whence, by compounding the two Proportions, $Sq^r. S. ACD : S. CAD \times S. ADC$: : $2PQ : AC$; from which AC is known.

Corol. 3.

Since it appears, that the Triangles ADB and EHI are equal and alike in all respects, and, therefore, the horizontal Distance AB, *universally*, equal to the Perpendicular HI, it is manifest, that, when

HI

HI is the greateſt poſſible, AB will alſo be the greateſt poſſible; in which Circumſtance AC (if the Angle FAE be given) will likewiſe be the greateſt poſſible: And this, it is evident, muſt be, when HI coincides with MG, or when the Angles HEA and HAE are equal (as in *Fig.* 7 and 8); at which time the Point D coincides with H; becauſe AD and EH are always equal to each other. Therefore, ſince, in this Caſe, HAE (HEA) is $= NAH$, it follows, that the Amplitude, on any inclined Plane AE, will be the greateſt poſſible, when the Line of Direction AH biſects the Angle made by the Plane and Zenith.

Corol. 4.

Hence the greateſt Amplitude on any inclined Plane may alſo be known; for the right-angled Triangles AOG and HOB, having $AO = HO$ and the Angle O common, are equal in all reſpects; and therefore, as Tang. of AHG (BAH the Angle of Elevation): Tang. of CHG (CAB the Plane's Inclination) :: $AG : GC$; whence $AC = AG \mp GC$ is alſo known.

Corol. 5.

Hence, alſo, if the greateſt Amplitude on an inclin'd Plane be given, the greateſt horizontal Amplitude may be determined: For, Radius : $S.BAC$ :: $AC : BC = CG =$ the Difference of the given, and the required, Amplitudes.

Corol. 6.

But if, inſtead of the Plane's Inclination, the perpendicular Height, or Depreſſion, of the Object be given; then, AC ($AG \mp BC$) being to BC, as Radius to

to the Sine of BAC, and Radius : Cotang. BAC : : $BC : AB$; the greateſt Diſtance AB, at which the Ball can poſſibly hit the Object, will from hence be given: which Diſtance (becauſe $AC = AG \mp BC$, and $AB^2 = \overline{AC + CB} \times \overline{AC - BC}$) will alſo be expreſſed by $\sqrt{AG \times \overline{AG \mp 2BC}}$. Hence the greateſt horizontal Amplitude of a Ball, projected from a given Height above the Plane of the Horizon is known: For ST (*Fig.* 8.) may here be ſuppoſed to repreſent the Plane of the Horizon, and SA the given Height; and then SC, being equal to AB, is given from above $= \sqrt{AG \times \overline{AG + 2BC}}$.

Corol. 7.

But, if the horizontal Diſtance AB be given, and it be required to find the greateſt Height the Ball can poſſibly reach in the Perpendicular BCD; we ſhall have HG (AB) : AG : : Radius : Tang. of the Elevation (BAH or AHG); and Radius : Tang. BAC ($2BAH \backsim 90°$) : : $AB : BC$; which therefore is known. But (becauſe $AC \pm BC = AG$, and $\overline{AC + CB} \times \overline{AC - CB} = AB^2$) the ſame will alſo be truly exhibited by $\frac{AG^2 \backsim AB^2}{2AG}$.

Corol. 8.

Laſtly, let the Height, or Depreſſion, of the Object be given, together with its Diſtance AB, to determine the Direction, and the leaſt *Impetus* poſſible, to hit the Object: Then $AB : BC$: : Radius : Tang. BAC; whence the Elevation BAH is known: And as Radius : Tang. AHG (BAH) : : MG (AB) : AG; whence the *Impetus* is alſo known.